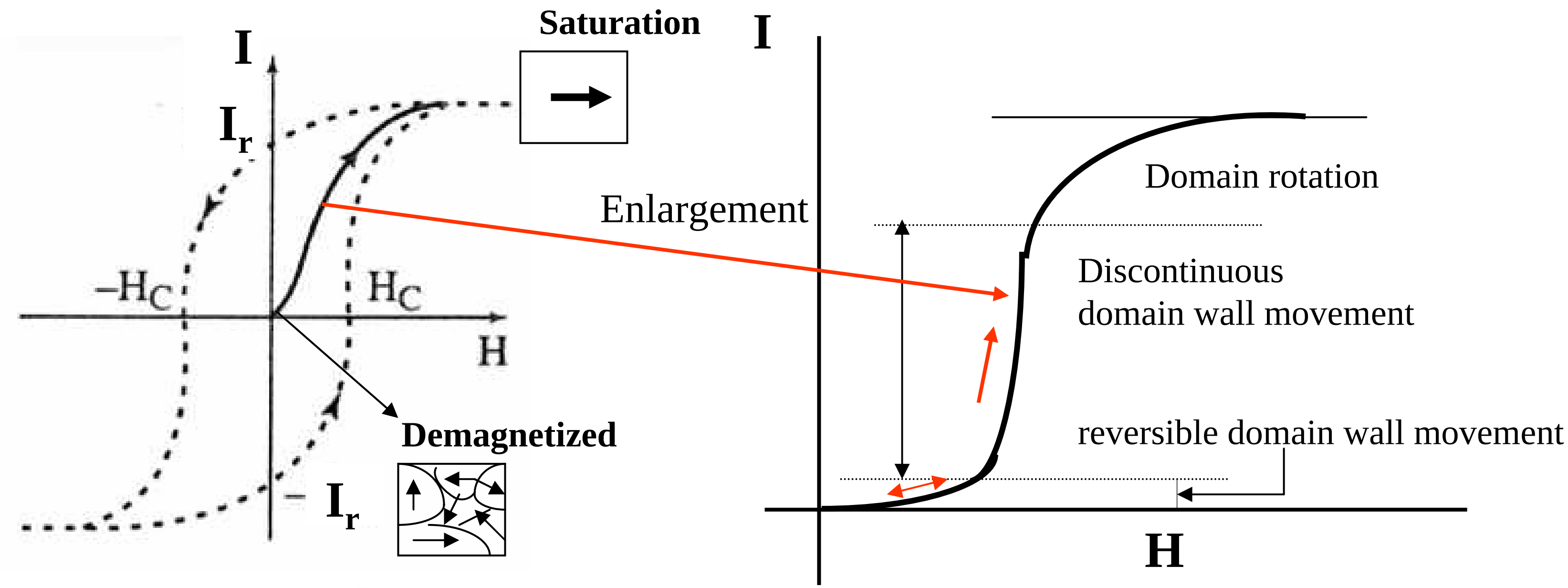
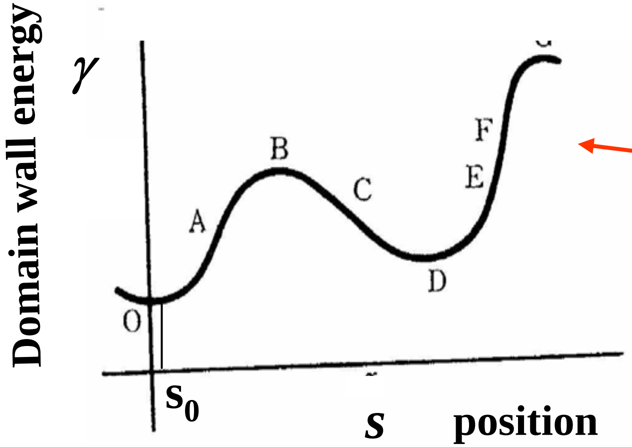
July 19th, 2006

2.4 Magnetization Process

Magnetization curve

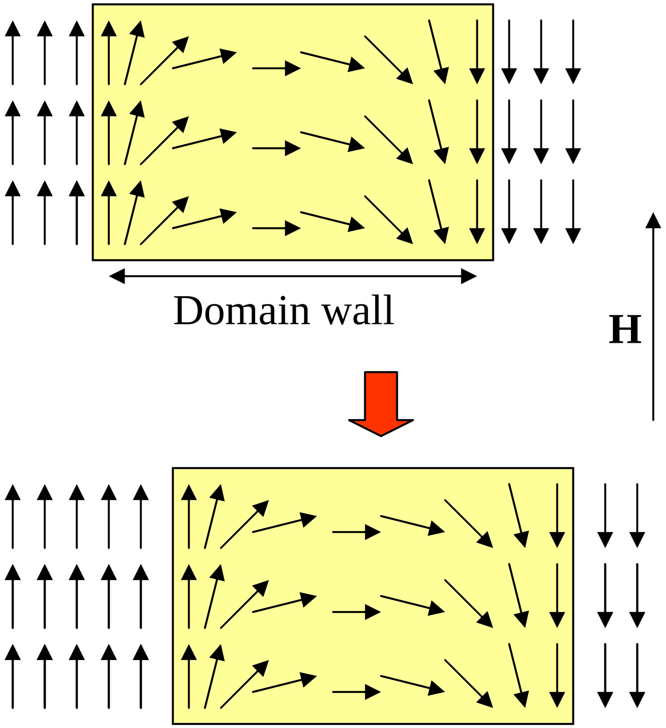


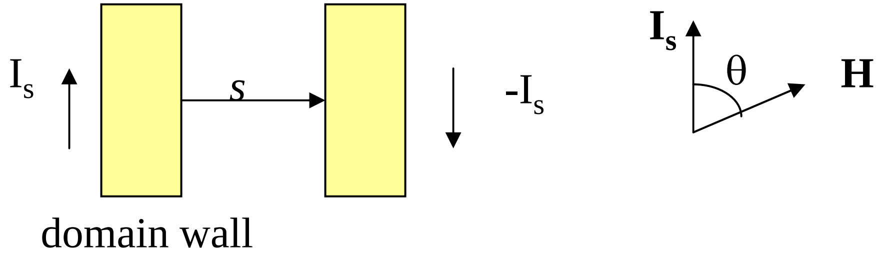


Materials are not uniform, which contain

- inclusions
- precipitates
- grain boundaries
- fluctuations in composition

Magnetization due to domain wall displacement





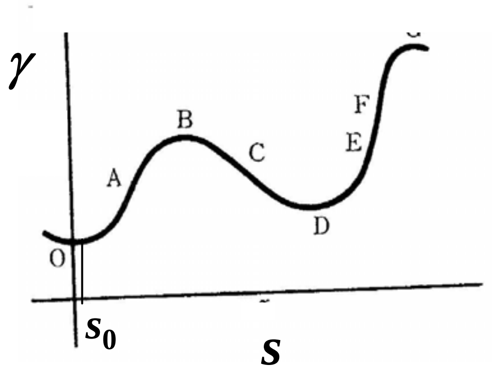
The movement distance of domain wall = s (m)
Area of domain wall = S

Increase of the magnetization is

$$\Delta I = 2 I_s S s \cos\theta \qquad (2.4.1)$$

Thus, the Zeeman energy by an external field is

$$E_H = -2 I_s H s \cos\theta \qquad (2.4.2)$$



If it is assumed that domain wall energy γ around s = s_0 is parabolic, γ is given by

$$\gamma = \frac{1}{2}\alpha s^2 \qquad (2.4.3)$$

Total energy is

$$E = \gamma + E_H = \frac{1}{2}\alpha s^2 - 2I_s H s\cos\theta$$

From $\partial E / \partial s = 0$ the distance of domain wall movement is given by

$$s = 2I_s H\cos\theta / \alpha$$

Thus, the increase of magnetization is

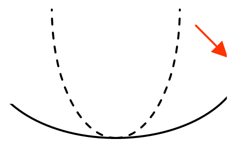
$$\Delta I = 2I_s S s\cos\theta = 4I_s^2 S H\cos^2\theta / \alpha$$

Initial susceptibility is

$$\chi_i = \Delta I / H = 4I_s^2 S\cos^2\theta / \alpha \qquad (2.4.4)$$

When magnetization is randomly orientated as in polycrystalline,

$$<\cos^2\theta> = \int_0^{\pi/2}\cos^2\theta\sin\theta d\theta = 1/3$$



Annealing

Thus, the susceptibility is

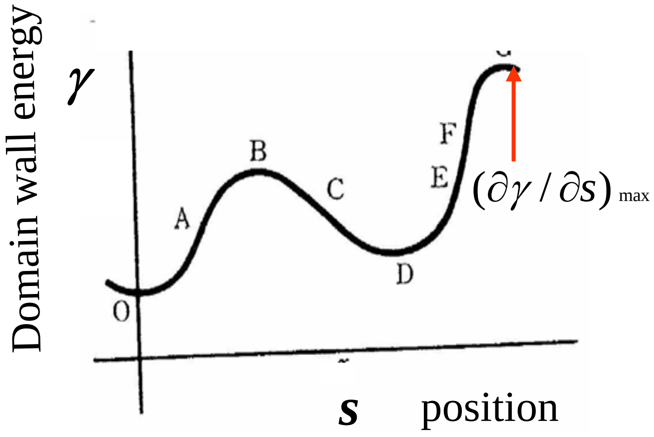
$$\chi_i = 4I_s^2 S / 3\alpha \qquad (2.4.5)$$

As $\alpha \propto \lambda, K$, the smaller (larger) λ, K induces soft (hard) magnetic property.

As α also depends on the defects, annealing for reducing the defects is effective for obtaining the soft magnetic properties.

Domain wall pinning and coercivity

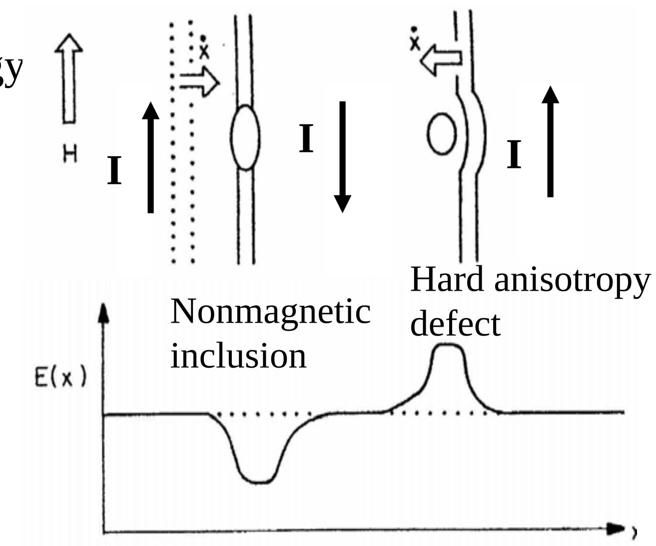
In real materials, domain walls do not move reversibly.



Grain boundaries
precipitates
inclusions
surface roughness
other defects

} lower the wall energy at a particular position in the material, effectively pinning its motion or they can place a barrier in front of the wall, inhibiting further wall motion through the defect.

Two classes of defects are shown in the following figure.
nonmagnetic inclusion (left) : reduces the local wall energy.
high-anisotropy defect (right): increases the local wall energy



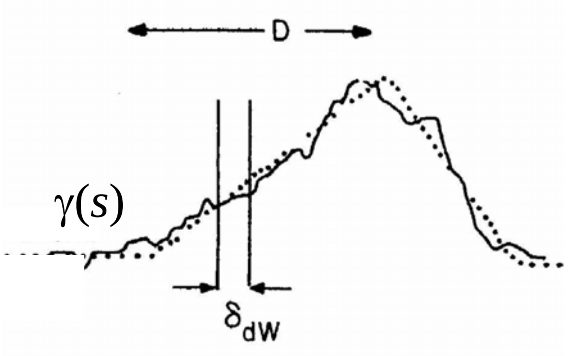
The steepest gradient in wall energy density can be taken to be **the magnetic pressure $2I_sH$** responsible for the coercivity;

$$(\partial\gamma/\partial s)_{max} = 2I_sH$$

$$H_c = (\partial\gamma/\partial s)_{max}/2I_s \qquad (2.4.6)$$

Quantitative models of domain wall motion generally distinguish two regions of behavior based on the ratio of **defect size D** to wall thickness $\delta_{dw} = \pi\sqrt{A/K}$

Consider the case of $D >> \delta_{dw}$ and materials parameter changes slowly over the length scale of the wall. In this case domain wall can be considered to be moving in an irregular but slowly changing potential.



The energy gradient $d\gamma / ds$ may be expressed as

$$\frac{d\gamma}{ds} = 4\frac{d}{ds}\sqrt{AK} = 2[\frac{\pi}{\delta_{dw}}\frac{\partial A}{\partial s} + \frac{\delta_{dw}}{\pi}(\frac{\partial K_{crys}}{\partial s} + \frac{3}{2}\lambda_s\sigma\frac{\partial\sigma}{\partial s})]$$

where $K_u = K_{crs} + \frac{3}{2}\lambda_s\sigma$ is used.

For $D >> \delta_{dw}$ we can approximate the gradient as linear, $\frac{d\gamma}{ds} = \frac{\Delta\gamma}{D}$

the coercive force $H_c = (\partial\gamma / \partial s)_{max} / 2I_s$ is expressed as

$$H_c \approx \frac{2H_a}{\pi}\frac{\delta_{dw}}{D}[\frac{\Delta A}{A} + \frac{\Delta K_{crys}}{K_{crys}} + \frac{3}{2}\lambda_s\frac{\Delta\sigma}{\sigma}] \qquad (2.4.6)$$

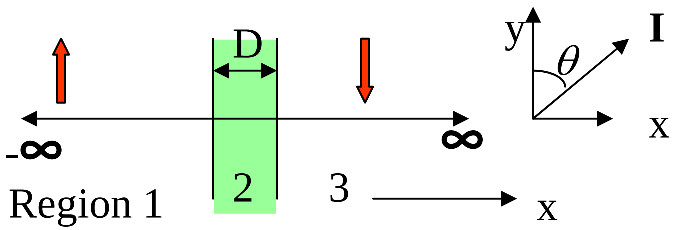
where $H_a = 2K / I_s$ is an anisotropy field.

Thus, the corecivity for $D >> \delta_{dw}$ in the gradual defect interface case becomes as δ_{dw}/D times a sum of fluctuation terms expressing local variations in exchange stiffness, crystal anisotropy and magnetoelastic energy, respectively.

Micromagnetic theory for well defined defects

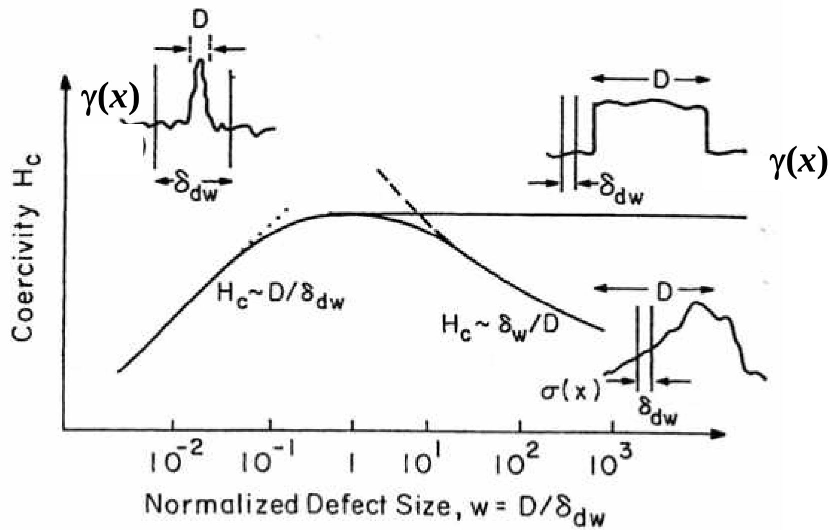
A micromagnetic theory of coercivity on domain wall pinning by <u>a sharply defined planner defect</u> has been developed, which consider the boundary conditions on the magnetization at the matrix/defect interfaces and takes exchange into account. <u>In this model the width D of the defect can be larger or smaller than the domain wall thickness</u>.

The energy density in this model includes micromagnetic contributions from **exchange energy**,**uniaxial anisotropy** and **Zeeman energy** densities. The angle θ, between the magnetization and the easy axis *y*, is a function of position for each term:

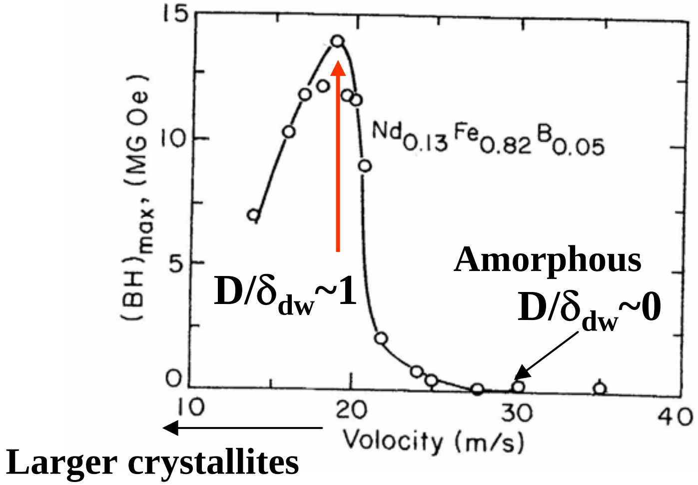


$$A_i\left(\frac{d\theta}{dx}\right)^2 + K_i \sin^2\theta - I_s H \cos\theta$$

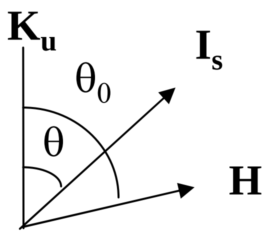
The **boundary conditions** are $d\theta/dx = 0$ at $\pm$ infinity and $\theta = 0$ and π at negative and positive infinity, respectively. The result is shown in the following figure.



Melt-spun NdFeB alloy



Magnetization due to domain rotation



Consider the single domain with an uniaxial magnetic anisotropy.

Magnetic energy is given by

$$E = K_u \sin^2\theta - I_s H \cos(\theta_0 - \theta) \qquad (2.4.7)$$

(1) *H* perpendicular to K_u (Hard-axis magnetization)

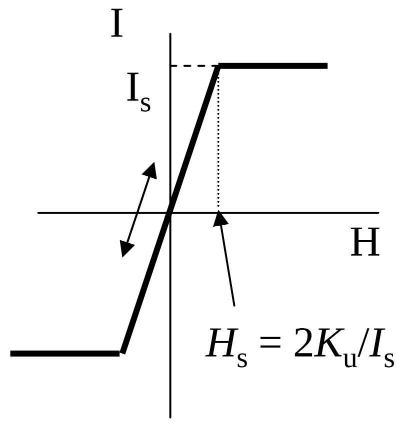
$$\partial E / \partial\theta = K_u \sin 2\theta - I_s H \sin(\theta_0 - \theta) = 0$$

For $\theta_0 = \pi / 2$

$$(2K_u \sin\theta - I_s H)\cos\theta = 0$$

Thus,

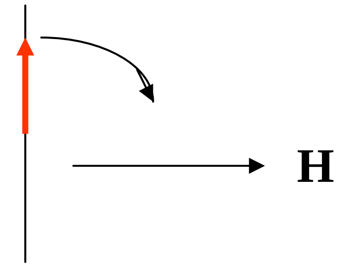
$$I = I_s \cos(\pi/2 - \theta) = I_s \sin\theta = I_s^2 H / 2K_u$$



$I = I_s$ for saturation field $\longrightarrow$ $\underline{H_s = 2K_u/I_s}$ (2.4.8)

Susceptibility is

$$\underline{\chi = I/H = I_s^2/2K_u} \qquad (2.4.9)$$



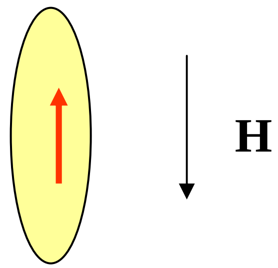
(2) *H* parallel to K_u (Easy-axis magnetization)

$$E = K_u \sin^2\theta - I_s H \cos\theta$$

$$\partial E / \partial\theta = K_u \sin 2\theta + I_s H \sin\theta = 0 \qquad (2.4.10)$$

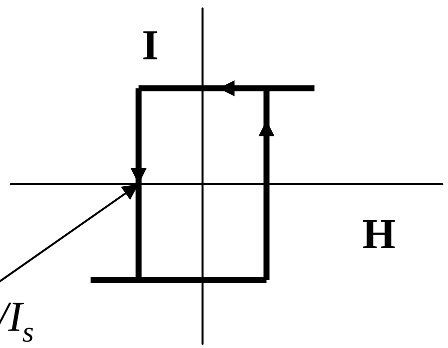
Stability condition is given by

$$\partial E^2 / \partial\theta^2 = 2K_u \cos 2\theta + I_s H \cos\theta > 0 \qquad (2.4.11)$$



Single domain

The solution of (2.4.10) is $\theta = 0$ and π.
From (2.4.11) $\theta = 0$ is stable only for $H > -2K_u/I_s$, while $\theta = \pi$ is stable only for $H < 2K_u/I_s$.
The solutions give a hysteresis curve shown in the following figure.

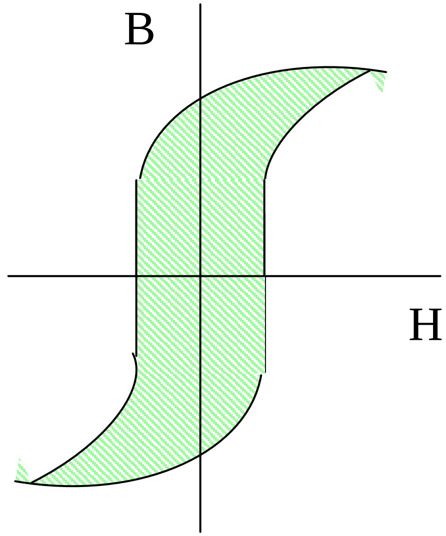


$H_c = 2K_u/I_s$

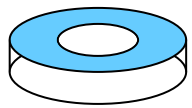
Square hysteresis loop

AC process

The area inside a B-H loop is exactly the energy per unit volume lost in one cycle of the hysteretic process, which will be verified in the following.



Toroidal core



Consider a toroidal core with area *A* and circuit length *l*.

Power loss over a period T is given by

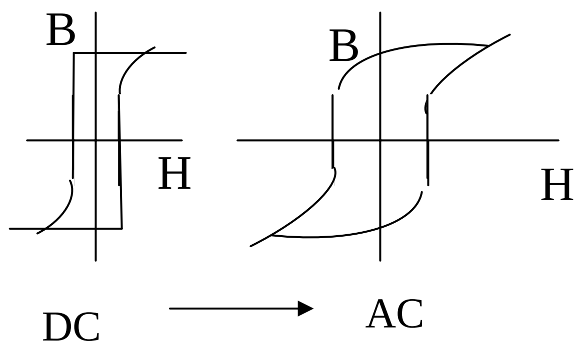
$$W = \int_{t=o}^{t=T} i(t)V(t)dt \qquad (2.4.12)$$

Ampere law gives $i = \oint Hds = lH$ and Faraday`s law gives $V(t) = -A\dfrac{dB}{dt}$

By inserting above both equations into (2.4.12), we obtain

$$W = lA\int_{t=o}^{t=T} H\frac{dB}{dt}dt = lA\oint H(t)dB \qquad (2.4.13)$$

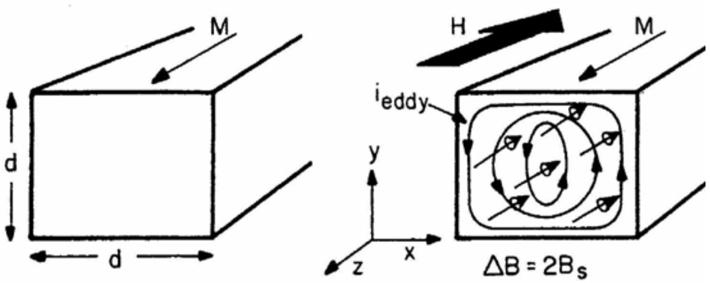
If the loop is traced at increasing frequency, it is observed that H_c increases and the loop becomes more rounded. The increase in loop area is a result of **eddy currents** induced in the sample by the flux density change.



Classical eddy-current loss

We will first assume **the magnetization changes uniformly** in the cross section of the material, There are no domain walls. This is the classical theory of eddy current loss.

The magnetization in a bar of square cross section is initially in the z direction and reverses uniformly under application of an external field H_z.



By Faraday's law the electric field induced around the direction of flux change is given by

$$(rot\mathbf{E})_z = -\frac{\partial \mathbf{B}}{\partial t} = i\omega B_m \qquad (B = B_m e^{-i\omega t})$$

$$(rot\mathbf{J})_z = i\omega B_m / \rho \qquad (J = E / \rho)$$

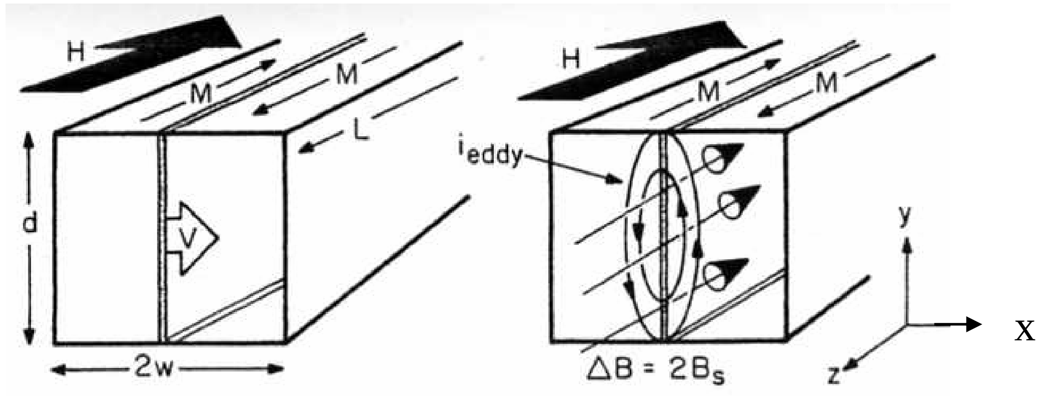
Considering of the symmetry of the problem

$$J_x = -Ay, J_y = Ax, \quad \text{with} \quad A = i\omega B_m / 2\rho$$

Time-averaged classical loss per unit volume is given by

$$P_{class} / V = \frac{1}{d^2} \iint J^2 \rho dxdy = \frac{4A^2}{d^2} \int_0^{d/2} \int_0^{d/2} (x^2 + y^2) dxdy = \underline{\omega^2 B_m{}^2 d^2 / 48\rho} \qquad (2.4.14)$$

Eddy-current loss about a single domain wall

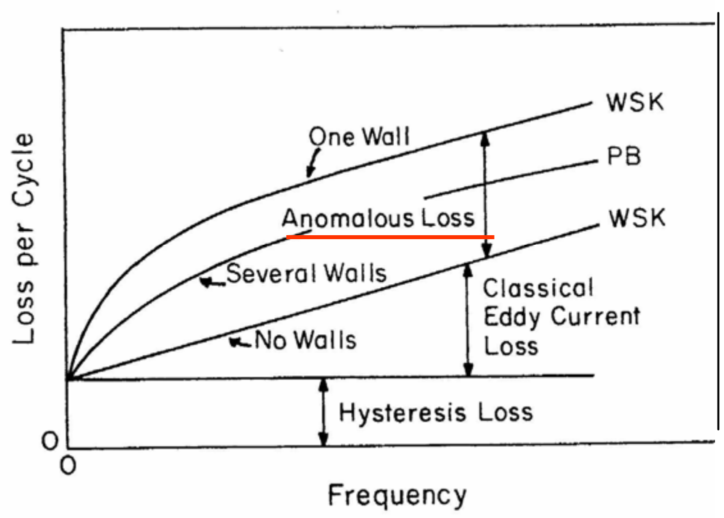


$$P_{micro}/V = \frac{P_{class}}{V}\frac{W_{AB}}{2\pi^2 d} \qquad (2.4.15)$$

W_{AB} is the distance a wall travels during a half-cycle.

Multiple domain walls

The introduction of more walls (1→N) decreases the wall spacing to W_{AB} = W/N, and hence is expected to lower the single-wall loss toward the classical value.



AC permeability and skin depth

When AC field is applied, flux density is retarded due to the eddy current loss.

$$H = H_0 e^{i\omega t} \qquad B = B_0 e^{i(\omega t - \delta)}$$

AC permeability is given by

$$\mu = B / H = B_0 e^{i(\omega t-\delta)} / H_0 e^{i\omega t} = (B_0 / H_0) e^{-i\delta} = (B_0 / H_0)(\cos\delta - i\sin\delta)$$

$$\mu = \mu' - \mu'', \qquad \mu' = (B_0 / H_0)\cos\delta, \qquad \mu'' = (B_0 / H_0)\sin\delta$$

AC permeability μ’ decreases compared with the dc value because of cos δ < 1.

The imaginary part μ’’ corresponds to energy loss, which is understood by the following.
The power loss energy per unit volume and unit time is given by

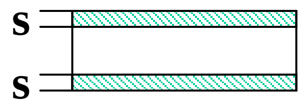
$$W = (\omega / 2\pi)\int_0^{2\pi/\omega} HdB = (\omega / 2\pi)\int_0^{2\pi/\omega} H(dB/dt)dt$$

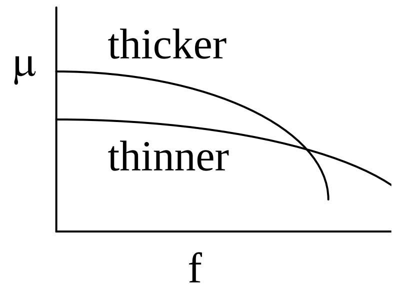
$$= (\omega / 2\pi)\int_0^{2\pi/\omega} [-\omega H_0 B_0 \cos(\omega t)\sin(\omega t - \delta)]dt = (\omega^2 / 2\pi) H_0 B_0 \int_0^{2\pi/\omega} \cos\omega t^2 \sin\delta dt$$

$$= \omega\ H_0 B_0 \sin\delta / 2 = \underline{\omega\mu'' H_0^{\ 2} / 2} \qquad (2.4.16)$$

$$\underline{\mu'' / \mu' = \tan\delta : \text{Loss factor}}$$

The penetration of time-dependent magnetic field into a metal is restricted due to skin effect. The **skin depth** is the the depth at which the magnetic field decays to 1/e of its value at surface.





$$\underline{s = (2\rho / \omega\mu\mu_0)^{1/2}} \qquad (2.4.17)$$

Microwave magnetization dynamics and ferromagnetic resonance

Under the magnetic field H magnetization precesses around the field;

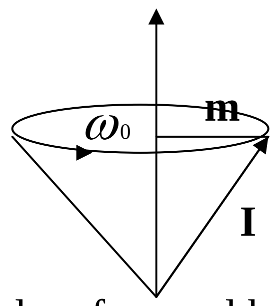
$$\frac{d\mathbf{I}}{dt} = -\gamma \mathrm{I} \times \mathrm{H} \qquad (2.4.18)$$

$$\gamma = g\frac{\mu_0 e}{2m_e} = g\mu_B/\hbar$$: Gyromagnetic ratio

The components are

$$\frac{dI_x}{dt} = -\gamma[I_y H_z - I_z H_y], \quad \frac{dI_y}{dt} = -\gamma[I_z H_x - I_x H_z], \quad \frac{dI_z}{dt} = -\gamma[I_x H_y - I_y H_x] \qquad (2.4.19)$$

When H is applied along z axis, the solution is



$$I_x = m\cos\omega_0 t, \quad I_y = m\sin\omega_0 t \quad I_z = const.$$

where $\omega_0 = \gamma H_z$ is the resonance frequency (**Larmor frequency**)
and m is the projection of I_x and I_y into the x-y plane.

In feromagnets there exist internal field (exchange field), which is of the order of several handred kOe. **Thus ω_0 ~ (1.105✕10^5)✕10^5 ~ 10^{10} Hz. (f ~ 10^9 Hz).**

Reraxation process

Equation (2.4.18) does not describe the fact that the moment eventually aligns with the field on the time scale of magnetometer measurement. What is missing is a **relaxation process**.

If it is assumed that the rate of relaxation is proportional to the amount by which the moment is out of equilibrium, Then the loss term is added to Eq. (2.4.18)

$$\frac{\partial I_z}{\partial t} = -\gamma (I \times H)_z - \frac{I_s - I_z}{\tau_1}$$

τ_1 **is the longitudinal relaxation time**. Similarly the transverse components relax toward zero, but with a different relaxation time;

$$\frac{\partial I_{x,y}}{\partial t} = -\gamma (I \times H)_{x,y} - \frac{I_{x,y}}{\tau_2}$$

Exactly, the magnetization dynamics is given by the LLG equation:

$$\frac{d\mathbf{I}}{dt} = -\gamma (\mathbf{I} \times \mathbf{H}) + \frac{\alpha}{I} \mathbf{I} \times \frac{d\mathbf{I}}{dt} \qquad (2.4.21)$$

where α is a damping factor.

What is the relation between α and τ ?

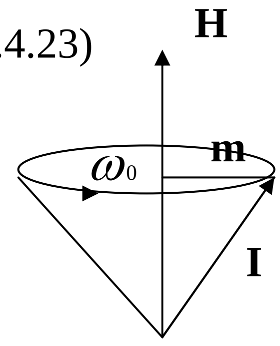
The components of (2.4.21) are given by

$$\frac{dI_x}{dt} = -\gamma I_y H_z + \frac{\alpha}{I}\left(I_y \frac{dI_z}{dt} - I_z \frac{dI_y}{dt}\right) \qquad \frac{dI_y}{dt} = \gamma I_x H_z + \frac{\alpha}{I}\left(I_z \frac{dI_x}{dt} - I_x \frac{dI_z}{dt}\right)$$

$$\frac{dI_z}{dt} = 0 + \frac{\alpha}{I}\left(I_x \frac{dI_y}{dt} - I_y \frac{dI_x}{dt}\right) \qquad (2.4.22)$$

If we assume that

$$I_x = m_x = m_0 e^{-t/\tau}\cos\omega_0 t \qquad I_y = m_y = m_0 e^{-t/\tau}\sin\omega_0 t \qquad (2.4.23)$$



By considering $I^2 = I_x^{\,2} + I_y^{\,2} + I_z^{\,2}$ we obtain

$$I_z = I[1-(m_0/I)^2 e^{-2t/\tau}]^{1/2} \qquad (2.4.24)$$

This expression means the magnetization I_z relaxes to I with the relaxation time τ.

Inserting (2.4.23) into (2.4.22), we obtain

$$\frac{dI_z}{dt} = \alpha m_0^{\,2}\omega_0 e^{-2t/\tau}$$

While (2.4.24) gives

$$\frac{dI_z}{dt} = \frac{(1/\tau)m_0^{\,2}e^{-2t/\tau}}{[1-(m_0/I)^2 e^{-2t/\tau}]^{1/2}}$$

From both the expression we obtain

$$\alpha\tau\omega_0 = \frac{1}{[1-(m_0/I)^2 e^{-2t/\tau}]^{1/2}} \qquad (2.4.25)$$

For $(m_0/I)^2 << 1$

$$\tau \approx 1/\alpha\omega_0 \qquad (2.4.26)$$

For ferromagnets α ~ 0.1, ω_0 ~ 10^{10} Hz,
τ ~ ns.

Electron spin resonance (Paramagnetic resonance)

The components of LLG eqs. (2.4.21) are given by

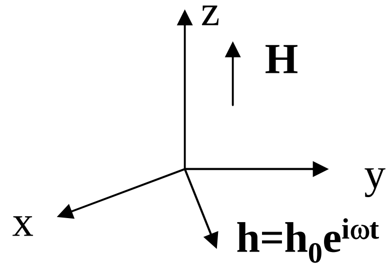
$$\frac{dI_x}{dt} = -\gamma[I_y H_z - I_z H_y] + \frac{\alpha}{I}\left(I_y \frac{dI_z}{dt} - I_z \frac{dI_y}{dt}\right)$$

$$\frac{dI_y}{dt} = -\gamma[I_z H_x - I_x H_z] + \frac{\alpha}{I}\left(I_z \frac{dI_x}{dt} - I_x \frac{dI_z}{dt}\right)$$

$$\frac{dI_z}{dt} = -\gamma[I_x H_y - I_y H_x] + \frac{\alpha}{I}\left(I_x \frac{dI_y}{dt} - I_y \frac{dI_x}{dt}\right)$$

$$\mathbf{I} = \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} m_x \\ m_y \\ I \end{pmatrix}$$

$$\mathbf{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} hx \\ hy \\ H \end{pmatrix}$$



If $m << I$, $h << H$, then the solution is

$$\begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \chi & -ik & 0 \\ ik & \chi & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_x \\ h_y \\ h_z \end{pmatrix} \qquad (2.4.27)$$

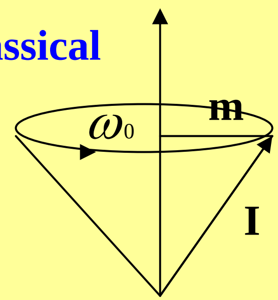
where

$$\chi = \frac{(\omega_0 + i\omega\alpha)\gamma H}{(\omega_0 + i\omega\alpha)^2 - \omega^2} \qquad (2.4.28)$$

$$\kappa = \frac{-\omega\gamma H}{(\omega_0 + i\omega\alpha)^2 - \omega^2} \qquad (2.4.29)$$

ESR

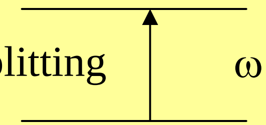
Classical



For $\omega = \omega_0$ the precession persists.

Quantum mechanical

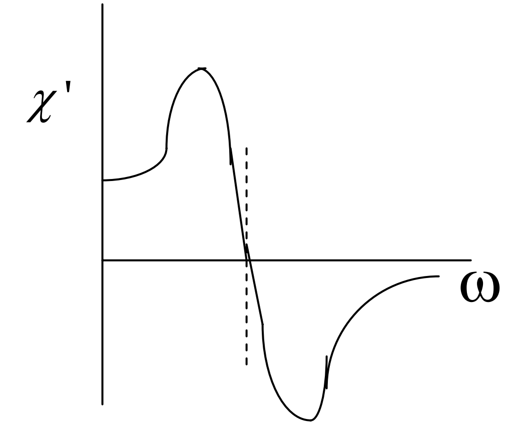
Zeeman splitting

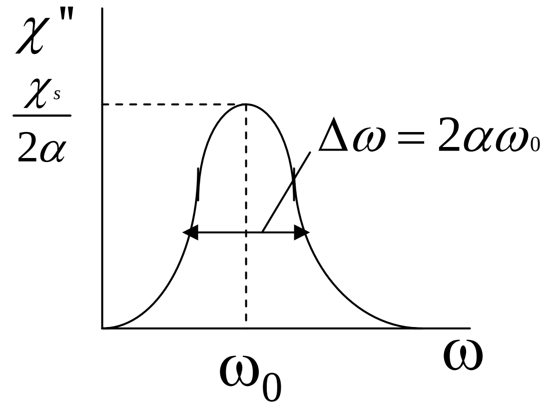


$\omega = \omega_0$

Eq. (2.4.28) gives $\chi = \chi' - j\chi''$

$$\left.\begin{aligned} \chi' &= \frac{\gamma I \omega_0(\omega_0^{\,2} - \omega^2) + \gamma I \omega^2 \alpha^2 \omega_0}{[\omega_0^{\,2} - \omega^2(1+\alpha^2)]^2 + 4\omega^2\omega_0^{\,2}\alpha^2} \\[2em] \chi'' &= \frac{\gamma I \omega\alpha[\omega_0^{\,2} + \omega^2(1+\alpha^2)]}{[\omega_0^{\,2} - \omega^2(1+\alpha^2)]^2 + 4\omega^2\omega_0^{\,2}\alpha^2} \end{aligned}\right\} \qquad (2.4.30)$$





For α < 1

$$\chi''(\omega = \omega_0) = \frac{\gamma I}{2\omega_0\ \alpha} = \frac{\chi_s}{2\alpha} \qquad \left(\omega_0 = \gamma H = \gamma \frac{I}{\chi_s}\ \right)$$

Maxmum half width

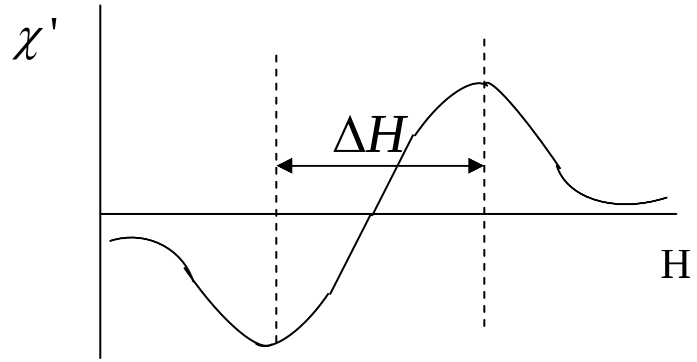
$$\Delta\omega = 2\alpha\omega_0 \qquad (2.4.31)$$

and

$$\gamma\Delta H = \Delta\omega \qquad \Delta H = \frac{2\alpha\omega_0}{\gamma}$$

From $\tau = \dfrac{1}{\alpha\omega_0}$ $\qquad \tau = \dfrac{2}{\gamma\Delta H}$ (2.4.32)

$$\chi'(\omega = \omega_0)\Delta H = I$$

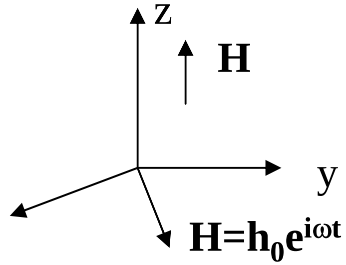


Ferromagnetic resonance

The resonance condition of the ferromagnets is altered by magnetostatic fields associated with the sampl shape. The magnetic field components become $H_i^j = H\text{-}N_j I_{j.,}$ j = x, y, z. In the limit of a weak microwave field $I_z \sim I_{s.}$ the resonance condition becomes

$$\frac{dI_x}{dt} = -\gamma[I_y(H_z - N_z I_s) + I_s N_y I_y] = -\gamma I_y[H + (N_y - N_z)I_s]$$

$$\frac{dI_y}{dt} = -\gamma I_x[H + (N_x - N_z)I_s]$$



Assuming the transverse magnetization components to vary as $e^{i\omega t}$, these equations are solved for

$$\omega^2 = \gamma^2[H + (N_x - N_z)I_s][H + (N_y - N_z)I_s] \qquad (2.4.33)$$

Knowing the sample shape, it is possible by a resonance experiment to determine a relaxation time from the line width and saturation magnetization or gyromagnetic ratio.

For thin films $N_x = N_y = 0$, and $N_z = 1$

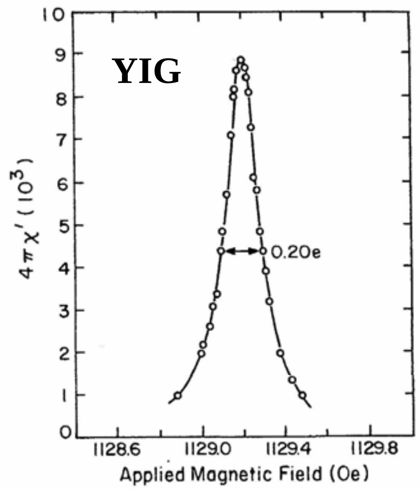


Figure 9.30. Ferromagnetic resonance line for a polished YIG sphere : ture in a microwave field of 3.33 GHz [after LeCraw and Spencer (19